

Math 16B: Homework 1

Due: July 2

1. For the following functions, find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial x \partial y}$:

(a) $f(x, y) = 3x^2 - 7xy^4$

(b) $f(x, y) = e^{xy} + y \ln(x)$

(c) $f(x, y) = \frac{3x^2 - 5}{y}$

2. A company's profits are given by $P(L, K) = 1000K\sqrt{L}$, where L labour and K is capital. If the company currently uses 81 units of labour and 169 units of capital, by approximately how much can they expect their profits to change if they increase labour by 2 units and decrease capital by 4?

3. Draw the level curves of the function $f(x, y) = x^2 - y^3$ at heights $-4, -1, 0$ and 3 .

4. Find all the relative maxima and minima, if any, of the following functions:

(a) $f(x, y) = x^3 - 2xy + 4y$

(b) $f(x, y) = ye^x - 3x - y + 5$

(c) $f(x, y) = x^3 + y^2 - 3x - 8y + 12$

5. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is 72 cm.

6. Three alleles (alternative versions of a gene) A, B, and O determine the four blood types A (AA or AO), B (BB or BO), O (OO), and AB. The Hardy-Weinberg Law states that the proportion of individuals in a population who carry two different alleles is

$$P = 2pq + 2pr + 2qr$$

where p , q and r are the proportions of A, B, and O in the population. Use the fact that $p + q + r = 1$ to show that P is at most $\frac{2}{3}$.

7. The base of an aquarium of volume 2 m^3 is made of slate and the sides are made of glass. If slate costs five times as much (per unit area) as glass, find the dimensions of the aquarium that minimize the cost of the materials.